GCSE Maths – Number

Terminating and Recurring Decimals

Notes

WORKSHEET

Decimals

Decimal points are used to separate **whole numbers** (integers) and **non-whole numbers**. For example, the decimal point in the number 5.7 separates the whole number, 5, from the non-whole part, 0.7.

The first column after the decimal point represents the **tenths**; the second represents the **hundredths**; the third represents the **thousandths** and so on. If we take the number 4.927, the whole number is 4, then we have 9 tenths, 2 hundredths and 7 thousandths.

A decimal number is just one way of expressing a number containing non-whole parts. Decimals can be converted into **percentages** and **fractions**, which are other ways of denoting non-whole numbers.

Terminating Decimals

A terminating decimal is one in which the **end of the numbers** beyond the decimal point is easy to identify. For example, 3.5 or 9.125 are terminating decimals because we can clearly see where the numbers end.

Terminating decimals can be **converted into fractions**. To do this, we must look at the numbers in each column (whole numbers, tenths, hundredths and so on). We write the numbers beyond the decimal point as the numerator (top of the fraction), and the appropriate factor of 10 as the denominator (bottom of the fraction).

Example: Convert 0.3 to a fraction.
In 0.3, there are 3 tenths. *This means that we can write it as $\frac{3}{10}$.*

If possible, we can then **simplify** or **cancel down** the fraction.

Example: Convert 0.375 to a fraction.
There are 3 tenths, 7 hundredths and 5 thousandths. *This can be written as $\frac{375}{1000}$.* *We can then try to simplify this fraction, which means making the numerator and denominator as small as possible by looking for the* ***highest common factor*** *of both.* *125 is the highest common factor of both 375 and 1000. Therefore, we divide both by 125.* *This gives us $\frac{3}{8}$.*

Decimal numbers larger than 1 can also be written as fractions, but the numerator will be larger than the denominator. However, we still follow the same method.

Example: Convert 4.6 to a fraction.

4.6 has 4 whole parts and 6 tenths.
We can write this as $\frac{46}{10}$.

Don't forget to simplify the fraction!
The highest common factor in this case is 2, so both 46 and 10 are divided by 2.
This leaves us with $\frac{23}{5}$.

We need to be able to convert back from fractions into decimals. This is done by simply dividing the numerator by the denominator.

Example: Convert $\frac{5}{8}$ into a decimal.

Using the 'bus stop' division method or another division method, we perform the calculation $5 \div 8$.

This gives us 0.625.

Again, the numerator can be larger than the denominator. We can still convert this type of fraction back into a decimal, it just means that it will be a **number larger than 1**.

Example: Convert $\frac{7}{2}$ into a decimal.

This is a simple division calculation.

$$7 \div 2 = 3.5$$

Recurring Decimals (Higher only)

As opposed to a terminating decimal, recurring decimals are ones in which the end of the numbers beyond the decimal point is **not identifiable**. In addition, there is a recurring element - the numbers **repeat infinitely** many times past the decimal point.

To represent a recurring decimal, a **small dot** is placed above the numbers that repeat (recur). For example, $2 \div 3 = 0.66666... = 0.\dot{6}$. This means that past the decimal point, the number 6 repeats infinitely many times.

There may be a **string of numbers** that recur past the decimal point. In this case, we see two dots - one above the **first number** in the **recurring series**, and one above the **last number** in the recurring series. For example,

$$1 \div 7 = 0.142857142857 \ldots = 0.\dot{1}4285\dot{7}$$

Converting a fraction to a recurring decimal is relatively straightforward. We use a division method to **divide the numerator by the denominator**, and then look for the number(s) that recur.

Example: Convert $\frac{13}{3}$ to a decimal.

Perform the division operation:

$$13 \div 3 = 4.333333 \ldots = 4.\dot{3}$$

Converting a recurring decimal to a fraction is slightly trickier. For this, we need to use **algebra**.

Example: Convert $0.\dot{8}\dot{2}$ into a fraction.

1. Write out an equation to show that an unknown fraction, x, is equal to the recurring decimal.

   Let $x = 0.82828282 \ldots$

2. We then write a second equation with the exact same recurring numbers after the decimal point.

   To do this, we multiply by a factor of 10*. In this case, we need to multiply each side by* 100 *to get the same recurring digits after the decimal point.*

   $$x = 0.82828282 \ldots$$
   $$100x = 82.82828282 \ldots$$

   The aim is to eliminate the recurring part of the number.

3. Subtract the first equation from the second.

   $$99x = 82.82828282 \ldots - 0.82828282 \ldots$$
   $$99x = 82$$

4. Solve for x.

   $$x = \frac{82}{99}$$

Example: Convert $0.\dot{3}2\dot{5}$ into a fraction.

1. Write out an equation to show that an unknown fraction, x, is equal to the recurring decimal.

$$\text{Let } x = 0.325325325\ldots$$

2. We then write a second equation with the exact same recurring numbers after the decimal point.

$$x = 0.325325325\ldots$$
$$1000x = 325.325325325\ldots$$

3. Subtract the first equation from the second.

$$999x = 325.325325325\ldots - 0.325325325\ldots$$

4. Solve for x.

$$x = \frac{325}{999}$$

Terminating and Recurring Decimals - Practice Questions

1) Calculate $0.75 + \frac{1}{5}$

2) Calculate $2.4 + \frac{3}{4}$

3) Calculate $0.25 + \frac{1}{4}$

4) Calculate $0.3 + 0.6$

5) Write $\frac{7}{9}$ as a recurring decimal.

6) Write $\frac{4}{3}$ as a recurring decimal.

7) Write $0.\dot{2}$ as a fraction.

8) Write $0.\dot{9}2\dot{3}$ as a fraction.

9) Write $3.\dot{1}\dot{6}$ as a fraction.

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.